Flexible Methodology for Image Segmentation

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Aim: One Framework to Segment Different Images

motion + Gaussian

Gamma noise

Poisson noise

MRA image

color image

color image

motion + Gaussian

60% pixel loss
+ Gaussian
Hyper-spectral Image and Point Cloud Classification

10% training pixels

Groundtruth
(98.88% accurate)

99.23% accurate

10% training
Outline

1. Mumford-Shah Model
2. Two-stage Image Segmentation Method
3. Three-stage Color Segmentation Model
4. Convex/Nonconvex Model
5. Hyperspectral Image and Point Cloud Classification
Given a corrupted image $f$, we want a $K$-phase segmentation to find a piecewise constant approximation with $K$ constant regions.

$\Omega \setminus \Gamma = \bigcup_i \Omega_i$

$g = c_i$ in $\Omega_i$, for $i = 1, 2, 3, 4$.
Mumford-Shah Model (1989) [cited 5,700+ time]

\[ \frac{1}{2} \int_{\Omega} (f - g)^2 \, dx \]

\[ + \frac{\mu}{2} \int_{\Omega \setminus \Gamma} |\nabla g|^2 \, dx \]

\[ + \text{Length}(\Gamma) \]

Data fidelity: control \( g \) not far away from \( f \)

Regularization: impose smoothness of \( g \) on \( \Omega \setminus \Gamma \)

Regularization: require boundary \( \Gamma \) be short

Highly non-convex problem
Simplifying Mumford-Shah Model

\[ E_{MS}(g, \Gamma) \]

Simplify it:
\[ \nabla g \equiv 0 \text{ on } \Omega \setminus \Gamma \]

\[ \frac{\lambda}{2} \int_{\Omega} (f - g)^2 \, dx \quad + \quad \frac{\mu}{2} \int_{\Omega \setminus \Gamma} |\nabla g|^2 \, dx \quad + \quad \text{Length}(\Gamma) \]

Multiphase Chan-Vese Model (02)
(minimizer \( \hat{g} \) is piecewise constant):

\[ E_{MS}(\{c_i\}, \Gamma) = \frac{\lambda}{2} \sum_{i=1}^{K} \int_{\Omega_i} (f - c_i)^2 + \text{Length}(\Gamma) \]
2-phase Chan-Vese Model (2001) [cited 10,200+ times]

\[
E_{CV}(\Gamma, c^i, c^e) = \frac{\lambda}{2} \int_{\Omega^i} (f - c^i)^2 + \frac{\lambda}{2} \int_{\Omega^e} (f - c^e)^2 + \text{Length}(\Gamma)
\]

Minimize by Alternating Direction Method:

Step 1: Given \(\Gamma\), minimize \(c^i\) and \(c^e\).
1. \(c^i = \text{average of } f \text{ in } \Omega^i\)
2. \(c^e = \text{average of } f \text{ in } \Omega^e\)

Step 2: Given \(c^i\) and \(c^e\), find \(\Gamma\).
Express \(\Gamma\) as zero-contour of a level set function \(\phi\) and minimize w.r.t. \(\phi\).
**How to Solve the Level-set Function?**

Minimizing $\phi$ in:

$$
\frac{\lambda}{2} \int_{\Omega} (f - c^i)^2 H(\phi) + \frac{\lambda}{2} \int_{\Omega} (f - c^e)^2 (1 - H(\phi)) + \int_{\Omega} \delta(\phi) |\nabla(\phi)|
$$

where $H(\cdot)$ is the Heaviside function, and $\delta = H'$. 

Euler-Lagrange equation for $\phi$:

$$
\delta(\phi) \left[ \text{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right) + \lambda (f - c^i)^2 - \lambda (f - c^e)^2 \right] = 0.
$$

Change to parabolic and solve by time-marching:

$$
\frac{\partial \phi}{\partial t} = \delta(\phi) \left[ \text{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right) + \lambda (f - c^i)^2 - \lambda (f - c^e)^2 \right].
$$
Convexified Chan-Vese Model

Smooth delta function $\delta(\cdot)$ by $\delta_\epsilon(\cdot)$:

$$\frac{\partial \phi}{\partial t} = \delta_\epsilon(\phi) \left[ \text{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right) + \lambda (f - c^i)^2 - \lambda (f - c^e)^2 \right].$$

T. Chan, Esedoglu, and Nikolova (06):

$$\hat{\phi} = \arg \min_{0 \leq \phi \leq 1} \left\{ \int_\Omega \left[ \frac{\lambda}{2} (f - c^i)^2 - \frac{\lambda}{2} (f - c^e)^2 \right] \phi(x) + \int_\Omega |\nabla \phi| \right\},$$

and $\Omega^i := \{x : \hat{\phi}(x) \geq \rho\}$ for a.e. $\rho \in [0, 1]$.

thresholding $\hat{\phi}$ by $\rho$ to get $\Omega^i$ and hence $\Gamma$
**Convexified Chan-Vese Model**

\[
E_{CV}(\Gamma, c^i, c^e) = \frac{\lambda}{2} \int_{\Omega^i} (f - c^i)^2 + \frac{\lambda}{2} \int_{\Omega^e} (f - c^e)^2 + \text{Length}(\Gamma)
\]

Minimize by Alternating Direction Method:

**Step 1:** Given \( \Gamma \), minimize \( c^i \) and \( c^e \).

1. \( c^i = \text{average of } f \text{ in } \Omega^i \)
2. \( c^e = \text{average of } f \text{ in } \Omega^e \)

**Step 2:** Given \( c^i \) and \( c^e \), find \( \Gamma \).

Express \( \Gamma \) as zero-contour of a level set function \( \phi \) and minimize w.r.t. \( \phi \).

**ADM:**
Many convex problems to solve

**Ours:**
Only 1 convex problem to solve

Convexify \( \phi \)  
(T. Chan et al. (06))
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Our Motivation

(a): True binary image

(b): Smooth image of (a)
Our Motivation

(d): Difference of (a) and (c) (nonzero pixel values only at the boundary)

piecewise-constant approximation

(d): Difference of (a) and (c) (nonzero pixel values only at the boundary)

thresholding smooth function
Aim in Segmentation

Chan-Vese: Get a piecewise constant approximation $\hat{g}$ of $f$

Our idea:

$f \rightarrow$ smooth approx. $\hat{g}$ of $f$ $\rightarrow$ threshold $\hat{g}$ $\rightarrow$ segmentation

stage 1 $\rightarrow$ stage 2
Stage One: Convex Variant of the M-S Model

\[ E_{MS}(g, \Gamma) \]

\[ \frac{\lambda}{2} \int_{\Omega} (f - g)^2 \, dx + \frac{\mu}{2} \int_{\Omega \setminus \Gamma} |\nabla g|^2 \, dx + \int_{\Gamma} |\nabla g| \, dx + \text{Length}(\Gamma) \]

Restrict: \( g \in W^{1,2}(\Omega) \)

\[ m(\Gamma) = 0 \]

Convex M-S Energy

\[ E(g) \]
Mumford-Shah Model for SBV

Explanation of the approximation:

For functions of bounded variations ([Ambrosio-Giorgi, 88]), Mumford-Shah model becomes

$$\min_{g \in SBV} \left\{ \frac{\lambda}{2} \int_{\Omega} |f - g|^2 + \frac{\mu}{2} \int_{\Omega \setminus J_g} |\nabla g|^2 + \mathcal{H}^1(J_g) \right\},$$

where $J_g$ is the jump set of $g$ and $\mathcal{H}^1$ is the Hausdorff measure of dimension 1.

- See [Cagnetti & Scardia, 08] and [Strekalovskiy et al., 12]
- If $g$ is binary and piecewise-constant, then $J_g = \Gamma$ and

$$\mathcal{H}^1(J_g) = \text{Length}(\Gamma) = \int_{\Omega} |\nabla g|$$
Convex Variant of the M-S Model

\[ E_{MS}(g, \Gamma) \]

\[ \frac{\lambda}{2} \int_{\Omega} (f - g)^2 \, dx + \frac{\mu}{2} \int_{\Omega \setminus \Gamma} |\nabla g|^2 \, dx + \text{Length}(\Gamma) \]

\[ \frac{\lambda}{2} \int_{\Omega} (f - g)^2 \, dx + \frac{\mu}{2} \int_{\Omega} |\nabla g|^2 \, dx + \int_{\Omega} |\nabla g| \, dx \]

Convex M-S Energy
\[ E(g) \]

Restrict:
\[ g \in W^{1,2}(\Omega) \]
Two-Stage Segmentation Method

\[
\frac{\lambda}{2} \int_{\Omega} (f - g)^2 \, dx + \frac{\mu}{2} \int_{\Omega} |\nabla g|^2 \, dx + \int_{\Omega} |\nabla g| \, dx
\]

Stage 1

Smooth solution \( \hat{g} \)

Stage 2

Threshold \( \hat{g} \) to piecewise constant
**Stage One: Extension to Blur/Projected Problems**

\[
\frac{\lambda}{2} \int_{\Omega} (f - g)^2 dx + \frac{\mu}{2} \int_{\Omega} |\nabla g|^2 dx + \int_{\Omega} |\nabla g| dx
\]

\[
f \leftarrow A g + n
\]

\[
\frac{\lambda}{2} \int_{\Omega} (f - Ag)^2 dx + \frac{\mu}{2} \int_{\Omega} |\nabla g|^2 dx + \int_{\Omega} |\nabla g| dx
\]

Extendable to images corrupted by blur or projection \(A\).

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**Unique Minimizer for Stage One**

Our *convex* variant of the Mumford-Shah model is:

\[
E(g) = \frac{\lambda}{2} \int_{\Omega} (f - Ag)^2 dx + \frac{\mu}{2} \int_{\Omega} |\nabla g|^2 dx + \int_{\Omega} |\nabla g| dx
\]

Its discrete version is:

\[
\frac{\lambda}{2} \| f - Ag \|_2^2 + \frac{\mu}{2} \| \nabla g \|_2^2 + \| \nabla g \|_1
\]

**Theorem**

Let \( \Omega \) be a bounded connected open subset of \( \mathbb{R}^2 \) with a Lipschitz boundary. Let \( \text{Ker}(A) \cap \text{Ker}(\nabla) = \{0\} \) and \( f \in L^2(\Omega) \), where \( A \) is a bounded linear operator from \( L^2(\Omega) \) to itself. Then \( E(g) \) has a unique minimizer \( \hat{g} \in W^{1,2}(\Omega) \).
Our Two-stage Segmentation Algorithm

Stage 1: solve $\hat{g}$ in

$$\min_g \left\{ \frac{\lambda}{2} \| f - \mathcal{A}g \|^2_2 + \frac{\mu}{2} \| \nabla g \|^2_2 + \| \nabla g \|_1 \right\}$$

Stage 2: determine threshold $\rho$ from $\hat{g}$ by mean or K-mean

Only 1 convex problem to solve

No iterations between Stages 1 and Stages 2

Given $f$

$\rho = 0.19$

$\hat{g} > 0.19$

$\hat{g} \leq 0.19$

$K$ phases
**Numerical Aspects: Stage One (Smoothing)**

Given $f$

**Stage 1:** solve $\hat{g}$ in

$$\min_g \left\{ \frac{\lambda}{2} \| f - Ag \|_2^2 + \frac{\mu}{2} \| \nabla g \|_2^2 + \| \nabla g \|_1 \right\}$$

solve by e.g. alternating direction method with multipliers (ADMM)

**Stage 2:** determine thresholds $\{\rho_i\}_{i=1}^{K-1}$ from $\hat{g}$

**$K$ phases**
**Numerical Aspects: Stage Two (Thresholding)**

Given $f$

Stage 1: solve $\hat{g}$ in

$$\min_g \left\{ \frac{\lambda}{2} \| f - Ag \|^2_2 + \frac{\mu}{2} \| \nabla g \|^2_2 + \| \nabla g \|_1 \right\}$$

Stage 2: determine thresholds $\{\rho_i\}_{i=1}^{K-1}$ from $\hat{g}$

$K$ phases

- 2-phase: mean of $\hat{g}$
- K-phase: use K-means on $\hat{g}$ to get $K$ clusters and their centroids $\{m_i\}_{i=1}^K$; then set thresholds

$$\rho_i = \frac{m_{i+1} + m_i}{2}$$

$\quad m_{i-1} \quad \rho_{i-1} \quad m_i \quad \rho_i \quad m_{i+1}$
Advantages of Smooth-&-threshold (SaT) Method

Given \( f \)

Stage 1: solve \( \hat{g} \) in
\[
\min_g \left\{ \frac{\lambda}{2} \| f - \mathcal{A}g \|_2^2 + \frac{\mu}{2} \| \nabla g \|_2^2 + \| \nabla g \|_1 \right\}
\]

Stage 2: determine thresholds \( \{\rho_i\}_{i=1}^{K-1} \) from \( \hat{g} \)

\( K \) phases

Advantages

- Stage 1 model for finding \( \hat{g} \) is convex
- Stage 2 uses the same \( \hat{g} \) when thresholds \( \rho_i \) or \( K \) change (No need to recompute \( \hat{g} \))
- No need to fix \( K \) at the very beginning
- Easily adapted to different kinds of corruptions (e.g. blur, projection, non-Gaussian noise)
2-phase image under vertical motion blur and Gaussian noise:

Given image  blurry & noisy image  Our solution $\hat{g}$
Motion Blurred and Noisy Image

Chan-Vese (01)  
Dong et al. (10)  
Yuan et al. (10)

$$\rho_{\text{mean}} = 0.7761$$  
$$\rho_{\text{user}} = 0.6$$  
$$\rho_{\{K=2\}} = 0.5048$$

Robust with respect to the thresholds chosen
4-phase Segmentation of Noisy and Blurry Image

Noisy & blurry

Yuan et al. (10)

Li et al. (10)

Sandberg et al. (10)

Steidl et al. (12)

Our 4 phases from \( \hat{g} \) using K-means \( \rho_i \)
Segmentation Changes with Threshold

\( \Gamma \) changes as \( \rho \) changes. But no need to solve for \( \hat{g} \) again. Just threshold \( \hat{g} \) to get the phases.
Tubular MRA Image: Stage 1 Solution

Given magnetic resonance angiography image

Our solution \( \hat{g} \)
Tubular MRA Image

Chan-Vese (01)  
Yuan et al. (10)  
$\rho_{\{K=2\}} = 0.4019$

Dong et al. (10)  
Cai et al. (13)  
$\rho_{\text{mean}} = 0.1760$

**Segmentation under Poisson or Gamma Noise**

First stage: given \( f \), solve

\[
\min_g \left\{ \lambda \int_{\Omega} (Ag - f \log Ag) dx + \frac{\mu}{2} \int_{\Omega} |\nabla g|^2 dx + \int_{\Omega} |\nabla g| dx \right\}.
\]

- **data fitting term** good for Poisson noise from MAP analysis
- also good for multiplicative Gamma noise (Steidl and Teuber (10))
- objective functional is convex (solved by Chambolle-Pock)
- admits **unique solution** \( \hat{g} \) if \( \text{Ker}(A) \cap \text{Ker}(\nabla) = \{0\} \)

Second stage: threshold the solution \( \hat{g} \) to get the phases.
3-object Image with Poisson Noise and Motion Blur

Original image

Noisy & blurred

Yuan et al. (10)

Dong et al. (10)

Sawatzky et al. (13)

$\rho_{\{K=2\}} = 129.94$
Fractal Tree with Gamma Noise and Gaussian Blur

Original image  Noisy & blurred  Yuan et al. (10)

Dong et al. (10)  Sawatzky et al. (13)  $\rho_{user} = 14$
Airplane with Multiplicative Gamma Noise

Original image  Noisy image

Yuan et al. (10)  Li et al. (10)  SaT with $\{\rho_i\}_{i=1}^2$ from K-means
4-phase with Close Intensity under Poisson Noise

Original image

Poisson noise

Yuan et al. (10)

Li et al. (10)

SaT with \( \{\rho_i\}_{i=1}^3 \) from K-means
Image with Close and Varying Intensities

Original image

Poisson noise

Yuan et al. (10)  Li et al. (10)  SaT with \( \{\rho_i\}_{i=1}^{3} \) from K-means
## CPU Time

**Multi-phase:** iteration numbers and CPU time in second

<table>
<thead>
<tr>
<th>Test</th>
<th>Yuan</th>
<th>Li</th>
<th>Our Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>iter.</td>
<td>time</td>
<td>iter.</td>
</tr>
<tr>
<td>Airplane</td>
<td>127</td>
<td>1.0</td>
<td>95</td>
</tr>
<tr>
<td>4-phase</td>
<td>57</td>
<td>2.2</td>
<td>49</td>
</tr>
<tr>
<td>Close-intensity</td>
<td>34</td>
<td>1.8</td>
<td>110</td>
</tr>
<tr>
<td>Varying-intensity</td>
<td>114</td>
<td>4.4</td>
<td>332</td>
</tr>
<tr>
<td>MRA</td>
<td>76</td>
<td>25.7</td>
<td>114</td>
</tr>
</tbody>
</table>

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1. Mumford-Shah Model
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3. Three-stage Color Segmentation Model
4. Convex/Nonconvex Model
5. Hyperspectral Image and Point Cloud Classification
Is 2-stage Enough for Color Images?

noisy image

K-mean thresholding

RGB: strong inter-channel correlation
Less-correlated Color Space

RGB: strong inter-channel correlation

Lab channels: less correlated

Thresholding using all six-channels
Three-stage (SLaT) Method for Color Images

Stage 1 (smoothing): given \( f = (f_1, f_2, f_3) \), solve

\[
\min_{g_i} \left\{ \lambda \int_{\Omega} (Ag_i - f_i)^2 \, dx + \frac{\mu}{2} \int_{\Omega} |\nabla g_i|^2 \, dx + \int_{\Omega} |\nabla g_i| \, dx \right\}, \ i = 1, 2, 3,
\]

to obtain smooth unique solution \( \hat{g} = (\hat{g}_1, \hat{g}_2, \hat{g}_3) \).

Stage 2 (lifting):

- transform \( \hat{g} \) to another color space \( \bar{g} = (\bar{g}_1, \bar{g}_2, \bar{g}_3) \) with less-correlation among the channels

- Then form the uplifted image \( g = (\hat{g}_1, \hat{g}_2, \hat{g}_3, \bar{g}_1, \bar{g}_2, \bar{g}_3) \)

Stage 3 (thresholding): Use K-means to threshold uplifted image \( g \) to get the phases.
## 2-phase Segmentation for Noisy Color Image

<table>
<thead>
<tr>
<th>Clean image</th>
<th>Noisy image</th>
<th>Li <em>et al.</em> (10)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Clean Image" /></td>
<td><img src="image2.png" alt="Noisy Image" /></td>
<td><img src="image3.png" alt="Li et al." /></td>
</tr>
<tr>
<td>Pock <em>et al.</em> (09)</td>
<td>Strorath <em>et al.</em> (14)</td>
<td>SaT with thresholds from K-means</td>
</tr>
<tr>
<td><img src="image4.png" alt="Pock et al." /></td>
<td><img src="image5.png" alt="Strorath et al." /></td>
<td><img src="image6.png" alt="SaT with thresholds" /></td>
</tr>
</tbody>
</table>

Gaussian noise with s.d. 0.1.
6-phase Segmentation for Noisy & Blurry Image

- Clean image
-blurry & noisy
-Li et al. (10)
-Pock et al. (09)
-Strorath et al. (14)
-SaT with thresholds from K-means

10-pixel vertical motion blur with Gaussian noise.
3-phase Segmentation for Noisy & Blurry Image

Clean image
blurry & noisy
Li et al. (10)
Pock et al. (09)
Strorath et al. (14)
SaT with thresholds from K-means

10-pixel vertical motion blur with Poisson noise added
4-phase Segmentation for Pixel-loss Color Image

Clean image
Noisy image
Li et al. (10)
Pock et al. (09)
Strorath et al. (14)
SaT with $\rho_{K=4}$

60% pixel loss with Poisson noise added

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Smooth-and-Threshold (SaT) Method

Convex M-S Energy
\[ E(g) \]

\[ \frac{\lambda}{2} \int_{\Omega} (f - Ag)^2 \, dx \]
\[ + \frac{\mu}{2} \int_{\Omega} |\nabla g|^2 \, dx \]
\[ + \int_{\Omega} |\nabla g| \, dx \]

Stage 1

Smooth solution \( \hat{g} \)

Stage 2

Threshold \( \hat{g} \) to piecewise constant
Relationship with Image Restoration

Convex M-S Energy

\[ E(g) \]

\[ \frac{\lambda}{2} \int_{\Omega} (f - Ag)^2 \, dx \]

\[ \frac{\mu}{2} \int_{\Omega} |\nabla g|^2 \, dx \]

\[ \int_{\Omega} |\nabla g| \, dx \]

reduce by introducing higher-order derivative:

T. Chan (00), Lysaker (03), Steidl (08), Bredies (10), Hintermüller (06), etc.

ROF Model (1992)
Rudin, Osher and Fatemi

staircase
Relationship with Image Restoration

\[ \frac{\lambda}{2} \int_{\Omega} (f - Ag)^2 \, dx + \frac{\mu}{2} \int_{\Omega} |\nabla g|^2 \, dx + \int_{\Omega} |\nabla g| \, dx \]

Hintermüller (2006): image restoration model

restoration + thresholding = segmentation

Cai & Steidl, EMMCVPR, 2013:

ROF model + thresholding = Chan-Vese
Non-convex Regularizers

Convex M-S Energy
\[ E(g) \]

\[ \frac{\lambda}{2} \int_{\Omega} (f - Ag)^2 dx \]  
\[ + \frac{\mu}{2} \int_{\Omega} |\nabla g|^2 dx \]  
\[ + \int_{\Omega} |\nabla g| dx \]

better restoration model

non-convex regularizer

better segmentation method

computational cost?
**Our Non-convex Regularizer**

\[
\phi(t; T, a) := \begin{cases} 
\frac{a(T_2-T)}{2T} t^2 & t \in [0, T) \\
-\frac{a}{2} t^2 + aT_2 t - \frac{aTT_2}{2} & t \in [T, T_2) \\
\frac{aT_2(T_2-T)}{2} & t \in [T_2, \infty)
\end{cases}
\]

with \( T_2 = T + \frac{1}{a^2} \)
Convex Non-convex Model

\[ \frac{\lambda}{2} \int_{\Omega} (f - g)^2 \, dx + \sum_{i=1}^{n} \phi (\| (\nabla g)_i \|_2; T, a) \]

- convex functional if \( \lambda > 9a \)
- non-convex regularizer

C., Lanza, Morigi, Sgallari, Num. Math., 2018
Two-Stage Convex Non-convex Segmentation

Convex Non-convex Energy
\[ \text{CNC}(g) \]

\[ \frac{\lambda}{2} \int_{\Omega} (f - g)^2 dx \quad + \quad \sum_{i=1}^{n} \phi (\| (\nabla g)_i \|_2; T, a) \]

\text{smooth solution } \hat{g} \text{ by ADMM}

\text{stage 2}

\text{Threshold } \hat{g} \text{ to piecewise constant}
Convex Non-convex SaT Segmentation Method

given image  Chan-Vese  Bae et al.  Dong et al.

Sandberg et al.  convex SaT + K-means  convex SaT with $\rho = .19$  convex non-convex SaT
4-phase with Close Intensity

Regions 1 to 4 by convex SaT

Regions 1 to 4 by convex non-convex SaT
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Hyper-spectral Image Classification

analyze the material for each pixel

Our method

Training pixels
10% = 1048 pixels
Our Smooth-and-Threshold (SaT) Approach
Indian Pines Data Set

- Data size: \(145 \times 145\) (spatial) \(\times\) 200 (spectral)
- Close spectrum between classes
Indian Pines Data Set

Error heat map over 10 trials with random 10% training pixels

<table>
<thead>
<tr>
<th>Method</th>
<th>Parameters</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ground-truth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SVM</td>
<td>[2, 5.98s]</td>
<td></td>
</tr>
<tr>
<td>SVM-CK</td>
<td>[3, 6.32s]</td>
<td></td>
</tr>
<tr>
<td>EPF</td>
<td>[4, 6.92s]</td>
<td></td>
</tr>
<tr>
<td>SC-MK</td>
<td>[9, 9.44s]</td>
<td></td>
</tr>
<tr>
<td>MFASR</td>
<td>[10, 443s]</td>
<td></td>
</tr>
<tr>
<td>SaT method</td>
<td>[5, 8.24s]</td>
<td></td>
</tr>
</tbody>
</table>

label color

[no. of parameters, time in seconds]
## Comparison with Other Methods

Accuracy over 10 random trials

<table>
<thead>
<tr>
<th></th>
<th>SVM</th>
<th>SVM-CK</th>
<th>EPF</th>
<th>SC-MK</th>
<th>MFASR</th>
<th>SaT</th>
<th>gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>overall accuracy</td>
<td>79.78%</td>
<td>92.11%</td>
<td>93.34%</td>
<td>97.83%</td>
<td>97.88%</td>
<td>98.83%</td>
<td>0.95%</td>
</tr>
<tr>
<td>average accuracy</td>
<td>80.11%</td>
<td>92.68%</td>
<td>95.95%</td>
<td>98.35%</td>
<td>97.91%</td>
<td>98.88%</td>
<td>0.35%</td>
</tr>
<tr>
<td>kappa</td>
<td>76.90%</td>
<td>91.01%</td>
<td>92.36%</td>
<td>97.52%</td>
<td>97.58%</td>
<td>98.66%</td>
<td>1.08%</td>
</tr>
</tbody>
</table>

- **overall accuracy**: percentage of correctly classified pixels
- **average accuracy**: average of the accuracy in each class
- **kappa**: Cohen’s kappa coefficient

SVM [Melgani et al., 2004], SVM-CK [Camps-Valls et al., 2006], EPF [Kang et al., 2014], SC-MK [Fang et al., 2015], MFASR [Fang et al., 2017].
### Effect of the High-order Smoothing Term

Accuracy over 10 random trials

<table>
<thead>
<tr>
<th></th>
<th>SaT with $|\nabla g|^2$</th>
<th>SaT without $|\nabla g|^2$</th>
<th>gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>overall accuracy</td>
<td>98.83%</td>
<td>97.26%</td>
<td>1.57%</td>
</tr>
<tr>
<td>average accuracy</td>
<td>98.88%</td>
<td>95.89%</td>
<td>2.99%</td>
</tr>
<tr>
<td>kappa</td>
<td>98.66%</td>
<td>96.86%</td>
<td>1.80%</td>
</tr>
</tbody>
</table>

- overall accuracy: percentage of correctly classified pixels
- average accuracy: average of the accuracy in each class
- kappa: Cohen’s kappa coefficient

Point Cloud Segmentation

10% training
Our Smooth-and-Threshold (SaT) Approach
**Result on 3-moon Segmentation**

Average accuracy over 10 random trials

<table>
<thead>
<tr>
<th>Method</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>MBO [2014]</td>
<td>99.12%</td>
</tr>
<tr>
<td>GL [2014]</td>
<td>98.4%</td>
</tr>
<tr>
<td>Ke I [2016]</td>
<td>98.4%</td>
</tr>
<tr>
<td>Ke II [2016]</td>
<td>98.6%</td>
</tr>
<tr>
<td>CVM [2017]</td>
<td>98.71%</td>
</tr>
<tr>
<td><strong>SaT method</strong></td>
<td><strong>99.23%</strong></td>
</tr>
</tbody>
</table>

| Gain          | 0.11%      |

MBO, GL [Garcia-Cardona et al., 2014],
Ke I, II [Ke & Tai, 2016],
CAM [Bae & Merkurjev, 2017]

Conclusions

- SaT (Smooth-and-Threshold) framework looks for smooth solutions before segmenting or classifying

- Convex segmentation model with unique solution—can be solved easily and fast

- Model solved only once—no need to solve the model again when threshold or number of phases changes

- Easily extendable to blurry images, non-Gaussian noise, image with information loss, color images, hyper-spectral images, and point cloud images

- Link image segmentation and image restoration
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- Alessandro Lanza, Serena Morini, and Fiorella Sgallari,
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- Gabriele Steidl, University of Kaiserslautern
References


Thank You!

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