



Fast Analog-to-Digital Compression for High Resolution Imaging

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Sampling: "Analog Girl in a Digital World..."



Limitations of Nyquist-Rate Sampling

- Large bandwidth requires high rate samplers
 - High rate communications
 - High resolution e.g. in radar and imaging
- High sampling rates lead to:
 - Large and expensive hardware-intensive systems
 - High-energy systems
 - Large digital databases that are difficult to process, store and transmit
- In medical imaging high rates often translate into long scanning times or high radiation dosages



ADCs, the front end of all digital devices, remain a major bottleneck

Q.: Can we recover information if we sample at a sub-Nyquist rate?







Super Resolution

- All measuring devices are bandwidth or resolution limited
- Abbe's diffraction limit in optical imaging: $DL = \frac{\lambda}{2NA}$ Spatial resolution is proportional to half the imaging wavelength



Spatial resolution in an antenna array or ultrasound probe is proportional to the aperture



Can we algorithmically recover the lost information using principles of sampling theory?

Xampling: Sub-Nyquist Sampling

Xampling = Compression + Sampling

- Reduce sampling rate in each ADC
- Reduce the number of elements used for imaging
 Main idea:



Substantial rate reduction and super resolution is possible!

Yonina C. Eldar

Sampling

Talk Outline

- Motivation for structure
- Xampling: Compression + sampling of analog signals
- Application to communication, radar and ultrasound
- Spatial subsampling
- Super resolution in microscopy and ultrasound
- Phase retrieval













Part 1: Motivation



Imaging Modalities

- Imaging in different frequency bands:
 - Spectral CT, hyperspectral imaging
- Radar and ultrasound imaging:
 - Multiple antennas
 - Streams of pulses
 - Beamforming

- Optical imaging: Lack of phase information

General framework for rate reduction and super resolution image recovery







Multiple Frequency Bands



- Can be viewed as f_{\max} -bandlimited
- But sampling at rate $\geq 2f_{\max}$ is a waste of resources
- For wideband applications Nyquist sampling may be infeasible

Streams of Pulses



Ultrasound



Processing Rates

- **I** To increase SNR and resolution an antenna array is used
- SNR and resolution are improved through beamforming by introducing appropriate time shifts to the received signals



- Requires high sampling rates and large data processing rates
- One image trace requires 128 samplers @ 20M, beamforming to 150 points, a total of 6.3x10⁶ sums/frame

Challenges

- Can we reduce analog sampling rates?
- Can we perform nonlinear beamforming on the sub-Nyquist samples without interpolating back to the high Nyquist-rate grid digitally?

Compressed Beamforming

Goal: reduce ultrasound machine size at same resolution



Enable 3D imaging Increase frame rate Enable remote wireless ultrasound





Subwavelength Imaging + Phase Retrieval

Collaboration with the groups of Moti Segev and Oren Cohen

- Diffraction limit: The resolution of any optical imaging system is limited by half the wavelength
- This results in image smearing
- Furthermore, optical devices only measure magnitude, not phase

λ=514nm



Nano-holes as seen in electronic microscope



Sketch of an optical microscope: the physics of EM waves acts as an ideal low-pass filter



Blurred image seen in optical microscope

Sparsity Based Subwavelength CDI

• Overcome both the lowpass filter and phase loss

Circles are 100 nm diameter Wavelength

532 nm



SEM image



Blurred image

Szameit et al., Nature Materials, 12

Sparse recovery



Diffraction-limited (low frequency) intensity measurements

Model Fourier transform

0

-5



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Xampling: Compression + Sampling

- Xampling: practical sub-Nyquist sampling and processing
- Many examples in which we reduce sampling rate by exploiting structure and goal
- Low rate translates to lower radiation dosage, faster scanning, processing wideband signals, smaller devices and improved resolution







Part 2: Xampling: Reduced rate sampling



Union of Subspaces



Allows to keep low dimension in the problem modelLow dimension translates to low sampling rate

Theorem

A sampling operator is invertible over a union of subspaces $\mathcal U$ if and only if it is invertible for every

$$\mathcal{A}_{\lambda,\gamma} = \mathcal{A}_{\lambda} + \mathcal{A}_{\gamma} = \{x | x = x_1 + x_2, \text{ where } x_1 \in \mathcal{A}_{\lambda}, x_2 \in \mathcal{A}_{\gamma}\}.$$

Xampling: Compression + Sampling

- Prior to analog sampling reduce bandwidth by projecting data onto low dimensional analog space
- Creates aliasing of the data
- Sample the data at low rate using standard ADCs in such a way that in the digital domain we get a CS problem
- Results in low rate, low bandwidth, simple hardware and low computational cost
- Achieves the Cramer-Rao bound given a sub-Nyquist sampling rate (Ben-Haim, Michaeli, and Eldar 12)
- Minimizes the worst-case capacity loss for a wide class of signal models (Chen, Eldar and Goldsmith 13)



Mishali and Eldar, 10







recovery

Xampling Hardware



The channels can be collapsed to a single channel











recovery

Compressed Sensing



- Sparse input vector with unknown support
- Sensing by sufficiently incoherent matrix (semi-random)
- Polynomial-time recovery algorithms from $K \log n$ measurements
- Modern optimization methods come into play

Compressed Sensing Extensions

- Nonlinear sparse recovery (optics):
 - Phase retrieval

(Shechtman et. al 11, 14, 15, Eldar and Mendelson 12, Ohlsson et. al 12)

Nonlinear compressed sensing

(Beck and Eldar 12, Bahman et. al 11)

- Reference based sparse recovery (MRI)
 (Weizman, Eldar and Ben Bashat 16)
- Sparsity with tracking (ultrasound) (Solomon et. al 18)
- Statistical sparsity (Solomon et. al 18, Cohen and Eldar 17,18)
- Deep learning from compressed samples









Analog Source Coding

Kipnis, Goldsmith and Eldar 17



- For a continuous signal x(t) and bit rate R what is the minimum possible distortion ?
- What is the minimum sampling rate that achieves this distortion?



Analog source coding theorem

$$R(f_s,\theta) = \frac{1}{2} \int_{-\frac{fs}{2}}^{\frac{fs}{2}} \log^+ \left[\tilde{S}_{X|Y}(f) / \theta \right] df$$
$$D(f_s,\theta) = mmse_{X|Y}(f_s) + \int_{-\frac{fs}{2}}^{\frac{fs}{2}} \min\{\tilde{S}_{X|Y}(f),\theta\} df$$

Optimal Sampling Rate

Shannon [1948]:



"we are not interested in exact transmission when we have a continuous source, but only in transmission to within a given tolerance"

Can we achieve D(R) by sampling below f_{Nyq}?



No optimality loss when sampling at sub-Nyquist (without input structure)!

Part 3: Applications to Comm, Radar and Ultrasound



"In theory, theory and practice are the same. In practice, they are not."

Albert Einstein

The Modulated Wideband Converter

Mishali and Eldar, 11





Spectral CT – with Pseudo-Polar Transform

Tsiper and Eldar 2018

- Separate materials based on spectral properties
- Transforming the spectral scan with RAPToR to the pseudo-polar grid
 - Better algebraic stability
 - Lower computational complexity
- New iterative algorithm for spectral decomposition
- Results out perform state-of-the-art in image quality



Original Phantom

Traditional Reconstruction SNR = 80dB



Our Method SNR = 40dB







Simulation Parameter	Value
Tube Voltage	120 keV
# Energy Bins	3
Basis Materials	Iodine, Calcium, Soft Tissue
SNR	40 dB

SPURS: SParse Uniform ReSampling

Kiperwas, Rosenfeld and Eldar 16

- When sampling non-uniformly one needs to interpolate to a uniform grid to apply FFT
- Particularly true when undersampling using non-Cartesian grids



- Conventional resampling is computationally demanding (NUFFT)
 (Fessler and Sutton 03)
- SPURS: Solution based on modern sampling methods and signal priors
- Efficient implementation using sparse system solvers and filtering

Conventional Beamforming

Non-linear scaling of the received signals

$$\Phi(t;\theta) = \frac{1}{M} \sum_{m=1}^{M} \varphi_m \left(\frac{1}{2} \left(t + \sqrt{t^2 - 4\gamma_m t \sin\theta + 4\gamma_m^2} \right) \right)$$

 γ_m - distance from *m*'th element to origin, normalized by *c*.

Performed digitally after sampling at sufficiently high rate

Individual traces $\varphi_m(t)$







Focusing along a certain axis – reflections originating from off-axis are attenuated (destructive interference pattern) SNR is improved

Compressed Beamforming

Each individual trace is buried in noise and has no structure

Fourier coefficient of

- Structure exists only after beamforming
- How can we perform beamforming on low rate data? How can we obtain small time shifts without interpolation?
- Compressed beamforming: Enables beamforming from low rate samples Key idea: Perform beamforming in frequency

$$c_{k} = \frac{1}{M} \sum_{m=1}^{M} \sum_{n} \varphi_{m}[n] Q_{k,m;\theta}[k-n]$$
Fourier coefficient

- of BMF signal signal at element *m* Logic: 1. BMF signal is a stream of pulses => can be
- recovered from a small number of $c_{k^2} \theta$ 2. Small: number of $c_{k^2} \theta$ (requires only a small) number of $\varphi_m[n]$

 $\exp \left\{ i \frac{2\pi}{rate} k \frac{\delta_m / c - t \sin \theta}{s ampling} \frac{\delta_m / c}{s mpling} \delta_m / c \right\}$



Chernyakova and Eldar, 14

Sub-Nyquist Ultrasound Imaging



Low rate sampling enables:

- **3**D imaging
- High frame rate for cardiac imaging
- Handheld wireless devices for rural medicine,
 - emergency imaging in the field/ambulance

Bring the Digital Revolution to Ultrasound, Anywhere

Xampling technology samples and processes ultrasound signals without loss of information at very low rates !

- Allows to integrate electronics into probe: wireless ultrasound
- Enabling an "open imager" advanced signal processing and ML methods that can run on any platform
- Enabling remote health flexibility



Demo Movie



Defense Applications

- Small, cheap radars with excellent resolution
- We can also reduce physical parameters:
 - Create a radar map in less time
 - Use fewer antenna elements
- Spectrum sharing between radar and communication over the same channel
- Free congested spectrum
- Fast frequency detection













Sub-Nyquist and Cognitive Radar



Part 4: Spatial subsampling

DAS - 63 Elements







Lateral Distance (mm

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Spatial CS in MIMO Radar

Rossi, Haimovich and Eldar, 13

- We can randomly dilute the number of elements in a MIMO radar using sub-Nyquist and CS ideas
- Using spatial Nyquist sampling the array aperture scales linearly with MN – the number of transmit and receive antennas
- Using CS we can get Nyquist resolution with MN scaling logarithmically with aperture





Hardware Prototype



Mishra et al., CoSeRa 2016

Combined temporal and spatial sampling is 12.5% of the Nyquist rate, same resolution

Mode 1			Y Tx	Rx	Parameters	Mode 1	Mode 2	Mode 3	Mode 4
					#Tx, #Rx	8,10	8,10	4,5	8,10
Mode 2					Element placement	Uniform	Random	Random	Random
	T STOPT				Equivalent aperture	8x10	8x10	8x10	20x20
Mode 3					Angular resolution (sine of DoA)	0.025	0.025	0.025	0.005
	~ ¶~¶ T				Range resolution	1.25 m			
Mode 4				0 0 0 Y	Signal bandwidth per Tx	12 MHz (15 MHz including guard-bands)			
		P T PP	pp c		Pulse width	4.2 μs			
	· • • • • • • • • • • • • • • • • • • •			ΥΥΥΥΥ	Carrier frequency	10 GHz			
0x20 ULA					Unambiguous range	15 km			
					Unambiguous DoA	180° (from -90° to 90°)			
	0 1	$1 \qquad 2 \qquad 3 \qquad 4$	4	5 6	PRI	100 µs			
			-	5 0	Pulses per CPI	10			
Aperture (m)				Unambiguous Doppler		from -75 m/s to 75 m/s			
	Prototyne Modes				Tech	Technical Specifications			

Antenna Selection via Deep Learning

- Standard approach is to choose elements at random
- Can we do better?

Deep learning:





Elbir, Mishra and Eldar, 2018



Antenna Selection for Imaging

Cohen and Eldar, 2018

- For imaging (radar, ultrasound) there isn't a single metric
- Image quality is determined by the beampattern *H*(θ) which represents the directivity of the beamformer and is given by



- Can we create the same beampattern with less elements?
- Factorization create beampattern by a convolution of two functions
 - In active sensing: convolve transmit and receive apertures
 - In passive sensing: convolve receive aperture with itself

Convolutional Beamforming

Convolutional Beamforming (COBA)

Output of a standard delay and sum beamformer:

 $b = \sum_{n=-N}^{N} z_n$ where z_n are the array signals after delays

- Convolutional beamformer (COBA):
 - Compute $y_n = \operatorname{sign}(z_n) \sqrt{|z_n|}$
 - Convolve s = y * y
 - Sum $b = \sum_{n=-2N}^{2N} s_n$
- The resulting beam pattern is

$$H(\theta) = \sum_{n=-2N}^{2N} \left(2N - |n| \right) e^{-2\pi j \frac{\sin(\theta)}{\lambda} nd}$$



Product Arrays

Beampattern of COBA can be written by the convolution theorem as

$$H(\theta) = \left(\sum_{n=-N}^{N} e^{-2\pi j \frac{\sin(\theta)}{\lambda} nd}\right) \left(\sum_{n=-N}^{N} e^{-2\pi j \frac{\sin(\theta)}{\lambda} nd}\right) = \sum_{n=-N}^{N} \sum_{m=-N}^{N} e^{-2\pi j \frac{\sin(\theta)}{\lambda} (\mathbf{n}+\mathbf{m}) dd}$$

Applying COBA is equivalent to a delay-and-sum on the sum co-array

$$S = \{n: n = i + j, i, j \in I\}$$

I – location set of the array

- Performance determined by the virtual co-array!
- The same sum co-array can be obtained from a sparse physical array where the number of elements is $2\sqrt{N} 3$.

Thus, the reduction factor is $(2N-1)/(2\sqrt{N}-3) \approx \sqrt{N}!$



In-Vivo Results



Part 5: Super-resolution in microscopy and US





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Super Resolution Microscopy

Solomon et. al 18

- Spatial resolution is proportional to half the imaging wavelength
- Noble prize 2014: super resolution using optical fluorescence microscopy (Betzig, Hell, Moerner)
- New measurement process control fluorescence of individual molecules
- Image the same area multiple times only a few point-emitters each time
- Superimpose the images
- Spatial resolution of ~20nm
- Limited temporal resolution!

> 10000 frames to collect all molecules

Can we get both high temporal resolution and high spatial resolution?







Correlation-Based Analysis

- For brightness images it is sufficient to estimate the variance of each pixel
- Power spectrum recovery can be performed at much lower rates than signal recovery!
- Translates into fewer images increasing temporal resolution

What is the minimal sampling rate to estimate the signal covariance?

- Assumption: Wide-sense stationary ergodic signal
- For covariance estimation we can substantially reduce the sampling rate even without structure!



Covariance Estimation

Cohen, Eldar and Leus 17

- Let x(t) be a wide-sense stationary ergodic signal
- We sample x(t) with a stable sampling set at times $\tilde{R} = \{t_i\}_{i \in \mathbb{Z}}$
- We want to estimate $r_x(\tau) = \mathbb{E}[x(t)x(t-\tau)]$

What is the minimal sampling rate to recover $r_x(\tau)$?

- Sub-Nyquist sampling is possible! Intuition:
- The covariance $r_x(\tau)$ is a function of the time lags $\tau = t_i t_i$
- To recover $r_x(\tau)$, we are interested in the difference set R:





 $t_i > t_i$

SPARCOM: SPARsity-Based Super **Resolution COrrelation Microscopy**

Solomon, Mutzafi, Segev, Eldar 18

- Take a small number of images with high density of fluorophores
- Compute correlations among images
- Fluorophores in different pixels are independent
- Recover image by estimating the pixel's variances on a high resolution grid

"Relax" the condition for a single emitter per diffraction limited spot

Mathematically:

• Acquired signal:
$$I(\mathbf{r}, t) = \sum_{k=1}^{L} u(\mathbf{r} - \mathbf{r}_k) s_k(t)$$

- In correlation space $R_I = AR_sA^H + N$
- Recovery based on Reweighted LASSO minimization

$$\min_{\boldsymbol{r}_{s}\geq0}\lambda||\boldsymbol{W}\boldsymbol{r}_{s}||_{1}+\frac{1}{2}\left\|\boldsymbol{R}_{l}-\sum_{l=1}^{N^{2}}a_{l}a_{l}^{H}\boldsymbol{r}_{s}^{l}\right\|_{F}^{2}$$

SPARCOM: Super Resolution Correlation Microscopy

Data taken from: http://bigwww.epfl.ch/smlm/datasets/index.html



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Super-resolution of T-cell Receptors

Collaboration with the group of Prof. Haran from Weizmann

- Immune response of T-cells involves T-cell receptor (TCRs) molecules
- TCRs are clustered inside the Leukocyte microvilli
 - **STORM** experiment with 30000 exposures
- SPARCOM performs reliable recovery with
 100 times shorter acquisition period



- Reconstruction of TCRs only at the focal plane!
- May lead to live cell inspection of TCR arrangement
 SPARCOM (green)
 STORM (red)
 Membrane topology



[nm]

Super Resolution Contrast Enhanced Ultrasound

Bar Zion et. al 18

Bolus injection of microbubbles into the blood stream Acquisition of consecutive frames to produce sub-wavelength image Micro-bubbles act as point emitters in the bloodstream



Temporal Mean



SUSHI: Sparsity-Based Ultrasound Superresolution Hemodynamic Imaging



Clinical evaluations with Dr. Anat Ilivitzki at Rambam

Exploiting Flow Dynamics

- Contrast agents flow is structured: within blood vessels
- Exploit flow to improve sparse recovery

Method:

- Combine sparse recovery with on-line tracking
- Use current locations as weights in sparse recovery
- Tracking by optical flow and Kalman filtering
- Yields estimate of contrast agents velocities

Diffraction limited

Frame-by-frame sparse recovery

Sparse recovery by exploiting flow

Solomon et. al 18



Velocities estimate









0

3

2 2 [mm/sec]

Super-resolution via Deep Learning

- Resolve overlapping UCAs via deep network scheme
 - Comparable performance to sparse recovery methods
 - Faster execution time
- Relies on the popular U-net architecture



Cost function:

```
||f(x|\theta) - G * y||_{2}^{2} + \lambda ||f(x|\theta)||_{1}

Parameters

Network

Supertresolved

frame
```

Small convolution kernel

Super-resolution of rat spinal cord vasculature



Phase Retrieval: Recover a signal from its Fourier magnitude

Collaboration with Profs. Moti Segev and Oren Cohen, Technion

Fourier + Absolute value

 \rightarrow $y[k] = |X[k]|^2$



- Arises in many fields: crystallography (Patterson 35), astronomy (Fienup 82), optical imaging (Millane 90), and more
- Given an optical image illuminated by coherent light, in the far field we obtain the image's Fourier transform
- Optical devices measure the photon flux, which is proportional to the magnitude

 $x[n] \rightarrow$

Phase retrieval can allow direct image recovery



Using ideas of structure and optimization we can enable phase recovery

Reviews

Recent overview:

Y. Shechtman, Y. C. Eldar, O. Cohen, H. N. Chapman, J. Miao, and M. Segev, "Phase retrieval with application to optical imaging," SP magazine 2015



Yoav Shechtman, Yonina C. Eldar, Oren Cohen, Henry N. Chapman, Jianwei Miao, and Mordechai Segev

Phase Retrieval with Application to Optical Imaging

Book chapters:

K. Jaganathan, Y. C. Eldar, and B. Hassibi, "Phase Retrieval: An Overview of Recent Developments," In *Optical Compressive Imaging* edited by A. Stern

T. Bendory, R. Beinert and Y. C. Eldar, "Fourier Phase Retrieval: Uniqueness and Algorithms", In *Compressed Sensing and its Applications: MATHEON Workshop*

Conclusions

- Sub-Nyquist sampling of many classes of signals
- Reduce scanning time and increase resolution by exploiting structure
- Structure also aids in robust interpolation
- Super resolution in microscopy and ultrasound using structure in the correlation domain
- Portable ultrasound based on sub-Nyquist techniques
- In the context of optics allows recovery of sub-wavelength info from optical far field and recovery from loss of phase





Can substantially reduce scanning times, device size, and increase imaging performance by carefully exploiting structure in analog and digital domains!

Future Vision



Exploit structure and goal



Xampling Website

webee.technion.ac.il/people/YoninaEldar/xampling_top.html

Y. C. Eldar, "Sampling Theory: Beyond Bandlimited Systems", Cambridge University Press, 2015



Y. C. Eldar and G. Kutyniok, "Compressed Sensing: Theory and Applications", Cambridge University Press, 2012 Yonina C. Eldar

SAMPL Lab Website



SAMPL Team



If you want to go fast go alone If you want to go far bring others

















If you found this interesting ... Looking for graduate students and post-docs!



